

Wrapped Branes and Supersymmetry

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Abstract

Configurations of two or more branes wrapping different homology cycles of space-time are considered and the amount of supersymmetry preserved is analysed, generalising the analysis of multiple branes in flat space. For $K3$ compactifications, these give the Type II or M theory origin of certain supersymmetric four-dimensional heterotic string solutions that fit into spin- $3/2$ multiplets and which become massless at certain points in moduli space. The interpretation of these BPS states and the possibility of supersymmetry enhancement are discussed.

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I. INTRODUCTION

Branes have played a crucial role in unraveling the non-perturbative structure of string theory and M theory [1]. In particular, p -branes which break half of the supersymmetry play a key role. They can be combined to form bound states and there are configurations in which they intersect or overlap either perpendicularly [2,3] or at angles [4–6] (For a review see, e.g., [7,8]). Both types of configurations, and in particular the amount of supersymmetry preserved by them, have been the subject of much study [9–11,3,5,12,8,13].

Branes can also wrap homology cycles of a compactifying space to give BPS states in the compactified theory that become massless when the cycle degenerates. When the BPS states that become massless fit into a vector multiplet, this results in an enhancement of the gauge symmetry [14–20].

The purpose of this paper is two-fold. First we wish to extend the study of multiple brane configurations and supersymmetry to investigate systems in which various branes wrap various *different* homology cycles. For toroidal backgrounds, the properties of such configurations follow immediately from the corresponding properties of intersecting or overlapping infinite branes in flat space, but for branes wrapping different cycles of $K3$ or a Calabi-Yau manifold, the situation is more complicated. In particular, it requires a suitable “alignment” between the calibrations of the various cycles. This is important for the study of symmetry enhancement with gauge groups of rank of at least two, resulting from two or more cycles degenerating simultaneously. We shall demonstrate the existence of supersymmetric, composite configurations, consisting of two branes wrapping round two different (shrinking) cycles of $K3$, with each brane separately breaking half of the original supersymmetry.

Second, we wish to investigate whether supersymmetry can be enhanced in the way that gauge symmetry can, with BPS states fitting into multiplets containing spin-3/2 becoming massless at special points in moduli space. In [16], such supersymmetry enhancement of, e.g., $N = 4$ supersymmetry in four dimensions ($D = 4$), was shown to be impossible without running into problems of higher spin or infinite numbers of massless fields. However, at enhanced symmetry points of the heterotic string with $N = 4$ supersymmetry in $D = 4$, e.g., the vector supermultiplets, containing the “W-bosons” that become massless are accompanied by an infinite number of magnetic and dyonic S-duality partners that also become massless [16], so that the result cannot be described by a field theory; in such exotic situations, the possibility of supersymmetry enhancement deserves re-examination. In [21,22], it was shown that there exist certain supersymmetric classical solutions of $N = 4$ supergravity coupled to 22 vector multiplets in $D = 4$ (effective theory of heterotic string compactified on T^6) that fit into multiplets containing spin-3/2 and whose BPS mass formula implies that they should become massless at special points in moduli space. These are configurations (and their S-duals) whose electric and magnetic charge vectors each have length-squared -2 . If the non-perturbative heterotic string indeed contains (quantum) BPS states with the charges carried by these configurations then these states fit into multiplets containing a gravitino that becomes massless at certain points in moduli space. This would then result in an enhancement of the supersymmetry. The points in moduli space at which this could happen [22] involve gauge groups of rank at least two. Examples of such massless configurations are associated with branes wrapping *two or more cycles* of $K3(\times T^2)$ which degenerate simultaneously, so that in this case the analysis involves studying various branes

wrapping various cycles, as discussed above. As is clear from the Type II description, each such $D = 4$ supersymmetric classical configuration is the coincidence limit of a two-centre solution and corresponds to the limit of a two-particle state in which two BPS states (with no mutual force) approach one another. If a bound state is formed, it must fit into a spin-3/2 multiplet which becomes massless at special points in the moduli space, which would result in supersymmetry enhancement. If no bound state is formed, there is no supersymmetry enhancement.

The equivalence [1] of Type IIA string theory compactified on $K3$, and heterotic string compactified on T^4 implies the existence of special points in the $K3$ moduli space at which the gauge symmetry is enhanced [15–18]. Indeed, it follows from supersymmetry that the mass of certain BPS states, preserving half of the supersymmetry, tends to zero at these special points [14,16] to give the extra massless vector multiplets. At these points the area of certain homology two-cycles shrinks to zero to give an orbifold limit of $K3$ and the BPS states in $D = 6$ arising from 2-branes wrapped round the shrinking two-cycles become massless [15,14,16].

The behaviour of the theory as a particular set of two-cycles shrinks can be studied by looking at the theory on the space $\mathbb{R}^{(D-5,1)} \times N_4$ in which the $K3$ is replaced by the appropriate asymptotically locally Euclidean (ALE) multi-instanton solution N_4 (e.g., for $SU(N)$ gauge symmetry, this is a multi-Eguchi-Hanson space). This space gives a good approximation when the radius of the two-sphere is small compared to the size of the $K3$. The enhanced gauge symmetry is then associated with parallel branes becoming coincident, so that homology two-spheres shrink to zero size and the membranes wrapped round these cycles give rise to massless states on the brane, whose world-volume theory thus becomes a non-Abelian gauge theory. This result is straightforward to check using duality; a T-duality relates this solution to a multi-centre solitonic 5-brane solution of Type IIB theory (in $\mathbb{R}^4/\mathbb{Z}_s$) and $SL(2, \mathbb{Z})$ duality relates this to a configuration of parallel 5-D-branes, and there is symmetry enhancement as these become coincident due to strings joining the 5-D-branes becoming massless [17].

Kaluza-Klein (KK) monopole space-times are again of the form $\mathbb{R}^{(D-5,1)} \times N_4$, but with N_4 now a $D = 4$ asymptotically locally flat (ALF) instanton solution. In [23], it was argued that KK monopoles can be regarded as branes (6-branes in M theory or 5-branes in $D = 10$ string theory) and that there is a similar symmetry enhancement as KK monopole branes become coincident. Again, in the coincident limit, certain two-cycles shrink to zero area and branes wrapping the cycles become null.

In this paper we shall generalise this to consider various branes wrapping various *different cycles* and analyse the surviving supersymmetry for such BPS configurations. We shall concentrate on ALE or ALF instantons, and the implications for $K3$ compactifications, although the generalisation to other spaces should be straightforward. In Section II we summarize the properties of the multi-instanton metric N_4 , the role it plays in repairing orbifold singularities of $K3$ and the conditions on cycles to be supersymmetric. In Section III we review the supersymmetry constraints for wrapped brane configurations. In Section IV we address multiple wrapped branes and the number of preserved supersymmetries. In Section V we relate the results in the Type II string picture to those in the dual heterotic string picture. In Section VI we discuss the possibility of enhanced supersymmetry.

II. CALIBRATIONS AND GRAVITATIONAL INSTANTONS

We shall restrict ourselves to supersymmetric p -branes which break half the supersymmetry when embedded as an infinite brane in flat space, or wrapped round a p -torus. The main example will be the D-branes of Type II string theory. If a p -brane is wrapped round an n -cycle ($n \leq p$) in a space-time N , then the amount of supersymmetry preserved depends on the cycle chosen. (The classification of supersymmetric cycles on Calabi-Yau spaces has been studied by several groups in Refs. [17,18].). A brane wrapped round a supersymmetric cycle preserves some fraction of the supersymmetry and the cycle is that of minimal n -volume in a given homology class. The volume form of the supersymmetric cycle is given by the pull-back of a certain closed form ω in N , which is a *calibration* of the cycle.

For a Calabi-Yau n -fold, there are n -dimensional supersymmetric cycles that are calibrated by $Re(\Omega)$, where Ω is the holomorphic $(n,0)$ -form, and these cycles are said to be special Lagrangian. There are also holomorphic $2m$ -cycles which are calibrated by $\frac{1}{m!}J^m$, where J is the Kähler form. A p -brane wrapped round a holomorphic cycle or a special Lagrangian cycle will preserve half the supersymmetry. For four-dimensional hyper-Kähler manifolds M , the supersymmetric two-cycles are the holomorphic two-cycles, i.e. those for which the volume form is proportional to the pull-back of one of complex structures of M .

ALE and ALF Spaces

The Gibbons-Hawking gravitational multi-instanton metric [24] is

$$ds^2 = V(dy + A_i dx^i)^2 + V^{-1} \delta_{ij} dx^i dx^j, \quad (1)$$

where $i = 1, 2, 3$ and

$$V^{-1} = \epsilon + \sum_{r=1}^s \frac{2n}{|x^i - x_r^i|}, \quad \nabla_i A_j - \nabla_j A_i = \epsilon_{ijk} \nabla^k V^{-1}. \quad (2)$$

If $\epsilon = 1$, this is the multi-Taub-NUT space with asymptotically locally flat (ALF) boundary conditions, while if $\epsilon = 0$, then n can be scaled to 1 and the space is a multi-centre generalisation of the Eguchi-Hanson metric with asymptotically locally Euclidean (ALE) boundary conditions. The case $\epsilon = 0, s = 1$ is flat space, while $\epsilon = 0, s = 2$ gives the Eguchi-Hanson instanton. The space $\mathbb{R}^{(D-5,1)} \times N_4$ where N_4 is an ALF or ALE instanton is a solution of M theory ($D = 11$) or of string theory ($D = 10$), respectively, as the gravitational instanton is hyper-Kähler [25]. It can be thought of as a set of $D - 5$ branes which will be referred to as G-branes, following [23].

The multi-Eguchi-Hanson space behaves as $\mathbb{R}^4/\mathbb{Z}_s$ at large distances, and can be used to resolve an A_{s-1} singularity of an orbifold limit of $K3$. For example, one orbifold limit of $K3$ is T^4/\mathbb{Z}_2 and each of the 16 orbifold singularities can be repaired by gluing in an Eguchi-Hanson metric. In the limit $|x_1 - x_2| \rightarrow 0$, the Eguchi-Hanson space becomes $\mathbb{R}^4/\mathbb{Z}_2$ with an orbifold singularity at the origin. While the multi-Taub-NUT solution can be thought of as a solution with a number of parallel $D - 5$ G-branes, the multi-Eguchi-Hanson space can be viewed as a number of parallel $D - 5$ G-branes in a transverse space which is an orbifold; as a result, not all the branes are independent as some are “mirror images” of others. There is one instanton corresponding to each pair $\{x_r, x_s\}$.

As the y coordinate is periodic, each line segment in the \mathbb{R}^3 parameterised by x^i is associated with a cylinder in N_4 , unless the line segment passes through one of the points x_r^i at which the size of the y -circle shrinks to zero. In particular, a line segment joining x_r^i and x_s^i corresponds to a two-sphere and the set of all such two-spheres corresponding to all pairs $\{x_r, x_s\}$ forms a basis for the second homology. As one approaches a point in moduli space at which $|x_r - x_s| \rightarrow 0$, the area of the corresponding two-sphere shrinks to zero. The minimal two-cycle (the one with least area) in the homology class, corresponding to the pair $\{x_r, x_s\}$, is given by the orbit of the straight line in \mathbb{R}^3 joining $\{x_r, x_s\}$ under the $U(1)$ generated by the Killing vector $\partial/\partial y$. For this to be a supersymmetric cycle, however, it is necessary that this cycle be holomorphic.

These hyper-Kähler spaces have three complex structures $(J^a)_i{}^j$ ($a = 1, 2, 3$) and three Kähler forms $J^a = \frac{1}{2}(J^a)_{ij}dx_\lambda^i dx^\lambda_j$ given by [26]

$$J^i = (dy + A_j dx^j) \wedge dx^i - \frac{1}{2} V \epsilon^{ijk} dx_\lambda^j dx^\lambda_k, \quad (3)$$

where the index a has been identified with the spatial index i . Consider the cycle corresponding to the pair $\{x_r, x_s\}$. Defining the normalised direction

$$n_{rs}^i = \frac{x_r^i - x_s^i}{|x_r^i - x_s^i|}, \quad (4)$$

the minimal cycle $\{x_r, x_s\}$ is holomorphic with respect to the complex structure [26]

$$J_{rs} = n_{rs}^i J_i. \quad (5)$$

In particular, the two-cycle $\{x_r, x_s\}$ and the two-cycle $\{x_t, x_u\}$ will only be holomorphic with respect to the *same* complex structure (5) if the corresponding line segments are parallel, i.e. $n_{rs}^i = \pm n_{tu}^i$.

III. WRAPPED BRANES AND SUPERSYMMETRY

Consider the space-time $\mathbb{R}^{(3,1)} \times N_4 \times T^2$ where N_4 is an ALE or ALF instanton with metric (1), regarded as a solution of Type IIA or Type IIB string theory. We choose coordinates X^M with $M = 0, \dots, 9$, which include the coordinates on N_4 , X^μ , $\mu = 1, \dots, 4$, with $X^4 = y$ and $X^i = x^i$ ($i = 1, 2, 3$). The coordinates on the two-torus T^2 are X^5, X^6 and the coordinates on the Minkowski space $\mathbb{R}^{(3,1)}$ are X^0, X^a with $a = 7, 8, 9$. The fact that N_4 is self-dual restricts the supersymmetry parameters ϵ to those satisfying

$$\Gamma_{N_4} \epsilon = \epsilon, \quad (6)$$

where Γ_{N_4} is the chirality projector on N_4 : $\Gamma_{N_4} \equiv \Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4$ and ϵ is a 32-component Majorana spinor in the type IIA case, and is a doublet of Majorana-Weyl spinors ϵ^i ($i = 1, 2$) of the same chirality for Type IIB string theory.

Consider a minimal two-sphere S_{rs} corresponding to the pair of points $\{x_r, x_s\}$. Coordinates on S_{rs} can be taken to be the coordinate $n_{rs}^i x^i$ along the line segment joining $\{x_r, x_s\}$,

together with y . For the type IIA theory, a 2-brane wrapped round S_{rs} will be preserved under those supersymmetries whose parameter satisfies

$$\Gamma_{(2)}\epsilon = \epsilon, \quad (7)$$

where

$$\Gamma_{(2)} = \frac{1}{36}\epsilon^{\alpha\beta\gamma}\partial_\alpha X^M\partial_\beta X^N\partial_\gamma X^C\Gamma_{MNP}, \quad (8)$$

and $\alpha = 0, 1, 2$ labels the 2-brane world-volume indices. Thus for the 2-brane,

$$\Gamma^0 n_{rs}^i \Gamma^i \Gamma^4 \epsilon = \epsilon. \quad (9)$$

This condition, together with (6), implies that the wrapped 2-brane preserves 1/4 of the original 32 Type II supersymmetries, or 1/2 of the 16 supersymmetries for Type II on $\mathbb{R}^{(3,1)} \times N_4 \times T^2$.

For a 4-brane wrapped round $S_{rs} \times T^2$, the condition (7) becomes

$$\Gamma^0 n_{rs}^i \Gamma^i \Gamma^4 \Gamma^5 \Gamma^6 \Gamma^{11} \epsilon = \epsilon. \quad (10)$$

For the type IIB string, a 3-brane wrapping $S_{rs} \times S^1$ (S^1 is along, say, X^5) the condition (7) becomes

$$\Gamma^0 n_{rs}^i \Gamma^i \Gamma^4 \Gamma^5 \epsilon^1 = \epsilon^2. \quad (11)$$

As before, the above two configurations also preserve 1/4 of the supersymmetry of the original Type II superstring, or 1/2 of the supersymmetry for Type II on $\mathbb{R}^{(3,1)} \times N_4 \times T^2$. It is straightforward to generalise the supersymmetry constraints to other branes.

For the Type IIA string, the dimensional reduction to $D = 4$ of the three-form C_{MNP} via the Ansatz $C = \sum_{rs} A_{\wedge}^{rs} X_{rs}$ gives a vector field A^{rs} corresponding to each two-cycle S_{rs} , where X_{rs} is the harmonic two-form dual to the two-cycle S_{rs} . A two-brane wrapped round S_{rs} gives a BPS state in $D = 4$ which preserves 1/4 of the original 32 Type IIA supersymmetries and is electrically charged with respect to A^{rs} , while the four-brane wrapped round $S_{rs} \times T^2$ gives a 0-brane that is magnetically charged with respect to A^{rs} . The mass of these BPS states depends on the area of S_{rs} (and the value of the two-form gauge field B_{MN} on N_4 [27]) and will vanish at a particular point in moduli space at which the area of S_{rs} vanishes. The electrically charged states correspond to the ‘‘W-boson’’ supermultiplets that become massless as $U(1)$ symmetry associated with A^{rs} is enhanced to $SU(2)$. At the same time the magnetic monopoles from the wrapped 4-branes also become massless [16]. Indeed, a whole $SL(2, \mathbb{Z})$ multiplet of dyons become massless at the same point of moduli space [16]. These arise as follows. In flat space, a 2-brane with charge q lying inside a 4-brane of charge p form a bound state that is a BPS state breaking half the supersymmetry if (p, q) are co-prime [9,10,28]. If the 2-brane is wrapped round S_{rs} and the 4-brane is wrapped round $S_{rs} \times T^2$, this gives rise to a BPS dyon in four dimensions with charge (p, q) preserving 1/4 of the original Type II 32 supersymmetries. These all become massless at the same point in moduli space when the area of S_{rs} tends to zero [16]. Alternatively, one can compactify first to 8 dimensions on T^2 , and wrap a $D = 8$ (p, q) dyonic membrane [28] round S_{rs} to get the same dyonic state when reducing further on N_4 .

IV. MULTIPLE BRANES

Intersecting Branes in Flat Space

There has been much interest in configurations of multiple branes that preserve some supersymmetry. A p -brane and a q -brane can intersect or overlap in an r -brane and be at an angle θ to one another. The amount of supersymmetry preserved depends on p, q, r and θ , and on the configuration of the D-brane world-volume Yang-Mills field [10,11]. For example, a 2-brane and a 4-brane of the Type IIA theory can intersect in a string. If they do so perpendicularly ($2 \perp 4$, $\theta = \pi/2$), the configuration preserves 1/4 of the supersymmetry [10,11,4,3,8]. For $\theta = 0$, so that the 2-brane lies inside the 4-brane ($2 \subset 4$), the (non-threshold) configuration preserves 1/2 of the supersymmetry, with a magnetic monopole configuration on the 4-brane [9,10,4]. The corresponding supergravity solution [28] is parameterised by an angle parameter $0 \leq \zeta \leq \pi/2$. For $\zeta = 0$ the solution is a single 4-brane, for $\zeta = \pi/2$ it is a 2-brane, while for general ζ it is a “dyon” carrying both 2-brane and 4-brane charge [28]. On reducing on T^2 , these give the dyonic membranes of [28], obtained by acting on an electrically charged membrane with a $U(1)$ subgroup of the $D = 8$ U-duality group, parameterised by the angle ζ .

For $\theta \neq \pi/2, 0$ the configuration still preserves 1/4 of the supersymmetry, provided additional background fields and/or branes are turned on. The corresponding supergravity solutions, which include $2 \perp 4$ and $2 \subset 4$ as special cases was given in [12]. The general case can be interpreted as a $(2 \perp 4|2)$ BPS (non-threshold) configuration¹. It is specified by two independent harmonic functions $H_{1,2}$ and the angle $\theta = \pi/2 - \zeta$. For $\theta = \pi/2$ it reduces to $2 \perp 4$ (1/4 of the supersymmetry) and for $\theta = 0$ it becomes a single 2-brane (It depends on one single harmonic function related to $H_{1,2}$). When one of the two original harmonic functions is turned off, i.e. $H_1 = 1$, the configuration (preserving 1/2 of the supersymmetry) has an interpretation as $2 \subset 4$ for all θ . (For $\theta = 0, \pi/2$ it becomes a single 2-brane and a single 4-brane, respectively.)

It will be useful to review the supersymmetry constraints for the $2 \perp 4$ configuration, which is specified by the harmonic functions $H_{1,2}$ for the 2-brane and 4-brane, respectively. Suppose that the 2-brane lies in the (X^1, X^4) plane, and the 4-brane lies in the (X^2, X^4, X^5, X^6) plane of the Minkowski space $\mathbb{R}^{(9,1)}$. The respective supersymmetry constraints for the 2-brane and 4-brane are [10,3,11]:

$$\begin{aligned}\Gamma^0 \Gamma^1 \Gamma^4 \epsilon &= \epsilon, \\ \Gamma^0 \Gamma^2 \Gamma^4 \Gamma^5 \Gamma^6 \Gamma^{11} \epsilon &= \epsilon.\end{aligned}$$

These constraints (12) are compatible because the constraints commute,

$$[\Gamma^0 \Gamma^1 \Gamma^4, \Gamma^0 \Gamma^2 \Gamma^4 \Gamma^5 \Gamma^6 \Gamma^{11}] = 0. \tag{12}$$

¹Here the notation $(2 \perp 4|2)$ denotes a composite configuration of two 2-branes and one 4-brane, with $2 \perp 4$ referring to the original configuration on which a specific boost and a subset of U-duality transformations were performed in order to arrive at the final configuration.

Thus the configuration $2 \perp 4$ is supersymmetric, preserving $1/4$ of the supersymmetry.

On the other hand, for the $2 \subset 4$ configuration with the 2-brane lying in the (X^1, X^2) plane, and the 4-brane in the (X^1, X^2, X^3, X^4) plane, the 2-brane and 4-brane supersymmetry constraints would be:

$$\begin{aligned}\Gamma^0 \Gamma^1 \Gamma^2 \epsilon &= \epsilon, \\ \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^4 \Gamma^{11} \epsilon &= \epsilon.\end{aligned}$$

In this case, the constraints anti-commute

$$\{\Gamma^0 \Gamma^1 \Gamma^2, \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^4 \Gamma^{11}\} = 0, \quad (13)$$

and the combined 2-brane and 4-brane bound state preserves the supersymmetries with parameters satisfying

$$\left(\cos \zeta \Gamma^0 \Gamma^1 \Gamma^2 + \sin \zeta \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^4 \Gamma^{11} \right) \epsilon = \epsilon. \quad (14)$$

There is one constraint and so the configuration breaks only half the supersymmetry [28].

For a general configuration $(2 \perp 4|2)$ with $H_1 \neq 1$ and $\theta \neq (0, \pi/2)$ the supersymmetry constraints are more complicated, involving additional non-zero background fields. Such a configuration can be obtained by starting with a known BPS configuration, e.g., $2 \perp 4$, and performing a set of boosts and U -duality transformations [12]. The original configuration has $1/4$ of the supersymmetry, and since the imposed transformations are consistent with the supersymmetry transformations, the resulting configuration continues to preserve $1/4$ of the supersymmetry.

We now extend this to (multiple) branes wrapped round homology cycles of some space. We will focus here on 2-branes and 4-branes on the space $N_4 \times T^2$ where N_4 is an ALE or ALF instanton, but the generalisation to other spaces and other branes should be straightforward.

Same Type p-branes Wrapping Different Homology Cycles

Consider first states arising from two 2-branes wrapped round two different cycles of $N_4 \times T^2$. One 2-brane is wrapped on S_{rs} and one wrapped on S_{tu} . Each is located at a particular point in $\mathbb{R}^{(3,1)} \times T^2$ and on compactification to $D = 4$ Minkowski space $\mathbb{R}^{(3,1)}$ give rise to two 0-branes located at two different points in $D = 4$. This gives two electrically charged “W-boson” vector supermultiplets, one of which is electrically charged with respect to A^{rs} and becomes massless as S_{rs} shrinks and the other of which is electrically charged with respect to A^{tu} and becomes massless as S_{tu} shrinks. This is a two-particle state, and no bound state is expected, even though there is no static force between the two multiplets. The amount of supersymmetry preserved depends on the complex structures of the two two-cycles. The one-particle state from a 2-brane on S_{rs} preserves half of the $N = 4$ supersymmetries if S_{rs} is holomorphic, while the 2-brane on S_{tu} also preserves half of the supersymmetry if S_{tu} is holomorphic; the combined two-particle state preserves half the $N = 4$ supersymmetries if S_{rs} and S_{tu} are holomorphic with respect to the *same* complex structure which, as explained in the previous section, requires the vectors $x_r - x_s$ and $x_t - x_u$ in \mathbb{R}^3 to be parallel. Choosing the two two-cycles to be holomorphic with aligned complex

structures allows a smooth deformation away from the enhanced symmetry point without any change in the amount of supersymmetry preserved.

Similarly, choosing two 4-branes, one wrapped on $S_{rs} \times T^2$ and one wrapped on $S_{tu} \times T^2$, gives a two-particle state with two magnetic monopoles and will preserve half the $N = 4$ supersymmetry if both S_{rs} and S_{tu} are holomorphic with respect to the same complex structure. Again, as both two-cycles shrink to zero, this configuration becomes massless.

The above examples of same-type p -branes ($p = 2, 4$) wrapping different cohomology cycles are supersymmetric configurations preserving half of the supersymmetry (provided the S_{rs} and S_{tu} are holomorphic with respect to the same complex structure), but no bound state is expected, even though there is no static force between the two multiplets. This is confirmed by the heterotic string picture. The first set of states corresponds to perturbative string states of toroidally compactified heterotic string with the charge lattice vector length-squared -4 , which are not in the spectrum of a single string (which requires length-squared not less than -2), but are in the two-string spectrum. The second set of states are (magnetic) \mathbb{Z}_2 S-duals of the perturbative string states and are thus also absent in the single string spectrum.

2-brane and 4-brane Wrapping Different Homology Cycles

Consider now the more interesting case in which a 2-brane is wrapped on S_{rs} and a 4-brane is wrapped on $S_{tu} \times T^2$, giving rise to a composite (two-particle) state in general. One particle is electrically charged with respect to A^{rs} and becomes massless as S_{rs} shrinks and the other is magnetically charged with respect to A^{tu} and becomes massless as S_{tu} shrinks. If S_{rs} is holomorphic, the 2-brane satisfies the supersymmetry constraints (6) and (9), while if S_{tu} is holomorphic the 4-brane preserves the supersymmetry constraints (6) and (10). Let the angle between the vectors $x_r - x_s$ and $x_t - x_u$ in \mathbb{R}^3 be θ . The analysis of the surviving supersymmetry is similar to that of a 2-brane and a 4-brane intersecting in a string at an angle θ . In particular, for $\theta = \pi/2$ the supersymmetry constraints (9) and (10) can be satisfied simultaneously, providing the S_{rs} and S_{tu} are holomorphic with respect to orthogonal complex structures which requires the vectors $x_r - x_s$ and $x_t - x_u$ in \mathbb{R}^3 to be orthogonal. This is because, in order to satisfy (9) and (10) simultaneously, the Γ matrices have to satisfy the following constraint:

$$[\Gamma^0 n_{rs}^i \Gamma^i \Gamma^4, \Gamma^0 n_{tu}^j \Gamma^j \Gamma^4 \Gamma^5 \Gamma^6 \Gamma^{11}] = 0, \quad (15)$$

which is satisfied if

$$\sum_{i=1}^3 n_{rs}^i n_{tu}^i = 0. \quad (16)$$

For say, $n_{rs} = (1, 0, 0)$ and $n_{tu} = (0, 1, 0)$, the supersymmetry constraints (9) and (10) are completely analogous to constraints (12) of a 2-brane and a 4-brane intersecting orthogonally in a string.

This analysis can be generalised to give dyonic two-particle states so that a 2-brane inside a 4-brane wraps $S_{rs} \times T^2$ to give a (p, q) dyon and another 2-brane inside a 4-brane wraps $S_{tu} \times T^2$ to give a (p', q') dyon. Alternatively one can start from $D = 8$ Type II theory on $\mathbb{R}^{(3,1)} \times N_4$ and wrap a (p, q) membrane on S_{rs} and a (p', q') membrane on S_{tu} . Note that

the supersymmetry constraints (9) and (10), are modified if one turns on background fields, and in this case it may be possible to satisfy these supersymmetry constraints also in the case when the vectors $x_r - x_s$ and $x_t - x_u$ in \mathbb{R}^3 point in directions that are not orthogonal.

V. COMPARISON WITH HETEROTIC STRING SOLUTIONS

The BPS states constructed above by wrapping branes on cycles of $N_4 \times T^2$, where N_4 is an ALE space, have analogues for branes on $K3 \times T^2$. Type II string theory compactified on $K3 \times T^2$ is equivalent to the heterotic string compactified on T^6 and the low-energy effective field theory is $N = 4$ supergravity coupled to 22 vector multiplets. The BPS 0-brane states can be associated with supersymmetric spherically symmetric supergravity solutions.

Perturbative states of the heterotic string carry electric charges $\vec{\alpha}$ that take values in an even self-dual lattice $\Lambda_{6,22}$ and have length-squared (with respect to the $O(6, 22, \mathbb{Z})$ norm) $\vec{\alpha}^T L \vec{\alpha} = -2, 0, 2, 4, \dots$. States carrying only magnetic charges $\vec{\beta}$ are related to these by S-duality, so that the magnetic charges also take values in the even self-dual lattice $\Lambda_{6,22}$ [29] and have length-squared $\vec{\beta}^T L \vec{\beta} = -2, 0, 2, 4, \dots$. States carrying magnetic charges are non-perturbative and arise from solitons or black hole solutions of the theory. The black holes in medium-short multiplets which become massless at certain points of moduli space have electric charge vector $\vec{\alpha}$ and magnetic charge vector $\vec{\beta}$ satisfying $\vec{\alpha}^T L \vec{\alpha} = \vec{\beta}^T L \vec{\beta} = -2$ [22] and the question arises as to whether these are single-particle quantum states of the non-perturbative heterotic string; if so, then there are gravitini that become massless, e.g., at the $SU(2) \times SU(2)$ or $SU(3)$ enhanced symmetry points [22] of the T^2 moduli subspace, so that as well as an enhancement of gauge symmetry, there is an enhancement of the supersymmetry. These medium-short multiplets fit into an infinite $SL(2, \mathbb{Z})$ representation, and all the $SL(2, \mathbb{Z})$ partners become massless at the same time as the original multiplet. Each contains a gravitino, so an infinite number of gravitini would become massless simultaneously in this scenario. Unfortunately we do not know the full physical state conditions on the electric and magnetic charges for the non-perturbative heterotic theory, which is why it is useful to compare these results with the dual Type II picture and use insights from the wrapped brane description of BPS states.

The BPS Mass Formula

The BPS mass formula, written in terms of states of heterotic string compactified on T^6 , is of the form [30,31]²:

$$M_{BPS}^2 = \frac{1}{2} e^{-2\phi_\infty} \vec{\beta}^T \mu_R \vec{\beta} + \frac{1}{2} e^{2\phi_\infty} \vec{\alpha}^T \mu_R \vec{\alpha} + \left[(\vec{\beta}^T \mu_R \vec{\beta}) (\vec{\alpha}^T \mu_R \vec{\alpha}) - (\vec{\beta}^T \mu_R \vec{\alpha})^2 \right]^{\frac{1}{2}}, \quad (17)$$

where

$$\vec{\tilde{\alpha}} \equiv \vec{\alpha} + \Psi_\infty \vec{\beta}, \quad \mu_{R,L} \equiv M_\infty \pm L. \quad (18)$$

² We use the notation and conventions, as specified in Refs. [30,21], following, e.g., [29].

The (quantised) charge vectors $\vec{\alpha}$ and $\vec{\beta}$ lie on a even self-dual (Narain) lattice $\Lambda_{6,22}$ [29]. The subscript ∞ refers to the asymptotic ($r \rightarrow \infty$) value of the corresponding fields. The moduli matrix M and the dilaton-axion field $S \equiv \Psi + ie^{-2\phi}$ transform covariantly (along with the charge vectors $\vec{\alpha}, \vec{\beta}$) under T-duality ($O(6, 22, \mathbb{Z})$) and S-duality ($SL(2, \mathbb{Z})$), while the BPS mass formula (17) remains invariant under these transformations. In the following we shall drop the ∞ subscript.

BPS States with 1/2 of the Supersymmetry

When the magnetic and electric charge vectors are parallel, $\vec{\beta} \propto \vec{\alpha}$, the BPS mass formula (17) is that of the BPS-saturated states which preserve 1/2 of $N = 4$ supersymmetry (see, e.g., [29]). These states fit into ultra-short multiplets, whose highest spin is one.

The electric charge vectors have length-squared $\vec{\alpha}^T L \vec{\alpha} = -2, 0, 2, 4, \dots$ with respect to the $O(6, 22)$ -invariant metric L . States carrying only magnetic charges are related to the electric states by S-duality and the charge quantisation condition then implies [29] that their magnetic charge vectors also lie on an even self-dual lattice $\Lambda_{6,22}$ with norm $\vec{\beta}^T L \vec{\beta} = -2, 0, 2, 4, \dots$. In addition there is an infinite number of dyonic states, preserving 1/2 of the supersymmetry, which are related to the purely electric states by S-duality. Their charge vectors are proportional, $\vec{\alpha} \propto \vec{\beta}$, with relatively co-prime electric and magnetic charges. Purely electrically charged states are in the perturbative string spectrum, while dyons and purely magnetically charged states are non-perturbative BPS configurations.

As classical supersymmetric solutions these states are spherically symmetric configurations. (i) Solutions with positive electric length-squared $\vec{\alpha}^T L \vec{\alpha} > 0$ (and its accompanying $SL(2, \mathbb{Z})$ tower) have null singularities, i.e. the horizon and the curvature singularity coincide, so that the horizon is singular. (ii) Solutions with $\vec{\alpha}^T L \vec{\alpha} = 0$ (along with its $SL(2, \mathbb{Z})$ tower) have naked singularities, so that a null probe reaches the singularity in a finite time as measured by an asymptotic observer, while the gravitational potential for massive test particles is attractive. For example, the well known KK monopole solution is in this class. These two classes of solutions have mass (17) which is always positive. (iii) Solutions with $\vec{\alpha}^T L \vec{\alpha} = -2$ (and the corresponding $SL(2, \mathbb{Z})$ tower) have typical naked singularities, i.e. a null probe again reaches the singularity in a finite time as measured by an asymptotic observer, but now the gravitational potential for massive test particles is repulsive³. They become massless at points of moduli space where $\vec{\alpha}^T \mu_R \vec{\alpha} = 0$. As these fit into vector multiplets, when they become massless, there is an enhancement of the gauge symmetry [16]. For example, at special points on the T^2 moduli subspace the gauge group enhancements are of the type $U(1)^4 \rightarrow U(1)^3 \times SU(2)$, $U(1)^4 \rightarrow U(1)^2 \times SU(2) \times SU(2)$ or $U(1)^4 \rightarrow U(1)^2 \times SU(3)$.

On the Type II string side the appearance of such massless states is due to wrapping of p -branes round shrinking two-cycles [14,16,17,19]. They can carry either an electric charge q , or a magnetic charge p , or can be dyonic with charge (p, q) . These dyonic BPS states

³ Qualitative features of these space-times are similar to those of the Schwarzschild black hole solutions with “negative mass” ($M < 0$). These solutions have naked singularities that repel massive test particles.

arise from a 2-brane with charge q wrapped round a two-cycle S of $K3$ and a 4-brane with charge p wrapped round $S \times T^2$ and, as they preserve half the $N = 4$ supersymmetry, they fit into ultra-short vector multiplets. In the quantum theory, the charges (p, q) are co-prime integers so that the states fill out an $SL(2, \mathbb{Z})$ representation. The purely electric states are the “W-bosons” that become massless when the two-cycle S degenerates and one of the $U(1)$ gauge symmetries is enhanced to $SU(2)$; at the same point in moduli space the infinite set of dyonic partners also become massless.

BPS States Preserving 1/4 of the Supersymmetry

In the case when the magnetic and electric charge vectors are not parallel, the the BPS mass is larger than that of BPS states preserving 1/2 of the supersymmetry (the last term in (17) is non-zero). These states are always non-perturbative and preserve only 1/4 of the $N = 4$ supersymmetry [30]. This means that the corresponding quantum states should fit into medium-short multiplets of $N = 4$ supersymmetry which include states of at least spin 3/2; it is expected that the multiplet includes scalars so that 3/2 is in fact the maximum spin.

The classical solutions correspond to the following types of spherically symmetric solutions. (i) Solutions with both charge norms positive, i.e. $\vec{\alpha}^T L \vec{\alpha} > 0$, $\vec{\beta}^T L \vec{\beta} > 0$, have regular horizons with *non-zero, moduli independent, Bekenstein-Hawking entropy* [31]: $\pi[(\vec{\alpha}^T L \vec{\alpha})(\vec{\beta}^T L \vec{\beta}) - (\vec{\alpha}^T L \vec{\beta})^2]^{1/2}$. (ii) Hybrid solutions with either electric or magnetic charge norm zero [negative] have null singularities [naked singularities]. (iii) Solutions with both electric and magnetic charge norms negative, i.e. $\vec{\alpha}^T L \vec{\alpha} = \vec{\beta}^T L \vec{\beta} = -2$, have again naked singularities, and in addition can become massless at special points of moduli space for which $\vec{\alpha}^T \mu_R \vec{\alpha} = 0$, $\vec{\beta}^T \mu_R \vec{\beta} = 0$, and $\vec{\beta}^T \mu_R \vec{\alpha} = 0$.

The latter set of solutions is of special interest. If there are corresponding quantum states with the same charge vectors, they must preserve 1/4 of the supersymmetry and so fit into spin-3/2 super-multiplets. Moreover, they must become massless at special points of moduli space, giving an infinite number of massless spin-3/2 quantum states and the possibility of supersymmetry enhancement [21].

For example, on a T^2 subspace, the moduli matrix M and $O(2, 2)$ invariant matrix L are of the form:

$$M = \begin{pmatrix} G^{-1} & -G^{-1}B \\ -B^T G^{-1} & G + B^T G^{-1}B \end{pmatrix}, \quad L = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}, \quad (19)$$

where $G \equiv [G_{mn}]$ ($(m, n) = 1, 2$), $B \equiv [B_{12}]$ are the four moduli on T^2 . The BPS mass formula (17) implies that these spin-3/2 multiplets become massless at points where $U(1)^4$ is enhanced to $U(1)^2 \times SU(2) \times SU(2)$ or $U(1)^2 \times SU(3)$ [22]. These two symmetry enhanced points take place at the point of moduli $(G_{11}, G_{22}, G_{12}, B) = (1, 1, 0, 0)$ (the self-dual point of the two-circle) and $(G_{11}, G_{22}, G_{12}, B) = (1, 1, \frac{1}{2}, \frac{1}{2})$, respectively. The massless dyonic solutions at these points of moduli space have the following respective charge vectors [22]:

$$(\vec{\alpha}, \vec{\beta}) = (\vec{\lambda}_{1\pm}, \vec{\lambda}_{2\pm}), \quad (20)$$

and

$$(\vec{\alpha}, \vec{\beta}) = (\vec{\lambda}_{i\pm}, \vec{\lambda}_{j\pm}), \quad [(i, j) = 2, 3, 4, \quad i < j], \quad (21)$$

where:

$$\vec{\lambda}_{1\pm} \equiv \pm(1, 0; -1, 0), \vec{\lambda}_{2\pm} \equiv \pm(0, 1; 0, -1), \vec{\lambda}_{3\pm} \equiv \pm(1, 1; -1, 0), \vec{\lambda}_{4\pm} \equiv \pm(1, 0; -1, 1). \quad (22)$$

Correspondence between the Dual Descriptions

The configurations described above arise from the Type II string on $K3 \times T^2$ as follows. Let S and S' be supersymmetric two-cycles of $K3$. Wrapping a 2-brane on S and a 4-brane on $S' \times T^2$ gives a $D = 4$ two-centre solution with one carrying the electric charge associated with 2-brane wrapping on S and the other carrying the magnetic charge associated with 4-brane wrapping on S' . These will preserve $1/4$ of the heterotic supersymmetry if the complex structures on S, S' are correlated appropriately, i.e the condition reduces to the orthogonality condition (15) in the limit of shrinking two-cycles of $K3$ in which the $K3$ can be approximated by an ALE space.

When the two centres coalesce (in $D = 4$), the dual description of the particular BPS solution emerges. E.g., choosing a particular hyper-surface of the T^2 : $(G_{11}, G_{22}, G_{12}, B) = (G_{11}, G_{22}, 0, 0)$, the BPS configurations with the dyonic charge vectors (20) have the BPS mass [21]:

$$M_{BPS} = e^\phi |G_{11}^{1/2} - G_{11}^{-1/2}| + e^{-\phi} |G_{22}^{1/2} - G_{22}^{-1/2}|. \quad (23)$$

Along this hyper-surface the configurations are massive, but become massless at $G_{11} = G_{22} = 1$ (the $SU(2) \times SU(2)$ point).

The Type II construction shows that the single-centre solution are in fact the coincident limit of two-centre solutions of the $D = 4$ heterotic supergravity theory. On the Type II string side we have a 2-brane and a 4-brane wrapping round different shrinking two-cycles, whose complex structures are orthogonal. The mass is proportional to the sum of the area of the two (shrinking) two-cycles. The comparison with the dual solution on heterotic string side above (i.e. along the specific hyper-surface $(G_{11}, G_{22}, 0, 0)$ with the BPS mass (23)), yields the following correspondence: the area of the two shrinking two-cycles is proportional to $|G_{11}^{1/2} - G_{11}^{-1/2}|$ and $|G_{22}^{1/2} - G_{22}^{-1/2}|$, respectively and the fact that the complex structures of the two two-spheres are orthogonal corresponds on the heterotic string side to the moduli choice $G_{12} = B = 0$.

At the $SU(3)$ point, on the heterotic string side we chose a particular hyper-surface with the moduli $(G_{11}, G_{22}, G_{12}, B) = (G, G, G/2, G/2)$. The twelve BPS states with dyonic charge vectors (21) all have the *same* BPS mass:

$$M_{BPS} = \left[\frac{2}{3}(e^{2\phi} + e^{-2\phi} + \sqrt{3}) \right]^{1/2} |G^{1/2} - G^{-1/2}|. \quad (24)$$

For $G = 1$ (the $SU(3)$ point), these configurations have zero mass.

On the Type II string side these twelve configurations correspond to 2-brane and 4-brane pairs, each wrapping round a different two-cycle, and the three two-cycles have complex structure directions that form an equilateral triangle. The area of each of the three two-cycles is thus the *same*; it in one to one correspondence with the value of $|G^{1/2} - G^{-1/2}|$ on

the heterotic string side. In the heterotic description, there is a two-form field on T^2 turned on, i.e. $B = G_{12} = G/2$, so that there should be a non-trivial two-form gauge field on the Type II side also.

VI. DISCUSSION: TWO-PARTICLE BPS STATES AND SUPERSYMMETRY ENHANCEMENT

We have seen that the $N = 4$ supergravity theory arising from $D = 4$ toroidally compactified heterotic string has (singular) black hole solutions that (i) carry electric charge with respect to one vector field and a magnetic charge with respect to another and (ii) preserve only $1/4$ of the $N = 4$ supersymmetry. This means that the corresponding quantum states should fit into medium-short multiplets of $N = 4$ supersymmetry which include states of at least spin $3/2$; it is expected that the multiplet includes scalars and $3/2$ is in fact the maximum spin. Moreover, the BPS mass formula implies that these multiplets become massless when the $U(1) \times U(1)$ with respect to which the solution is charged gets enhanced to $SU(2) \times SU(2)$ or $SU(3)$. This led to the conjecture that the supersymmetry could be enhanced at these points in moduli space, due to the spin- $3/2$ states in these BPS multiplets becoming massless [21]. In fact, there would be an infinite number of extra massless gravitini corresponding to the possible values of the electric and magnetic charges. The key question is whether these are single-particle (quantum) states of the non-perturbative heterotic string; unfortunately we do not know the full physical state conditions on the electric and magnetic charge vectors for the non-perturbative theory ⁴.

These $D = 4$ solutions are limiting cases of two-centre solutions in which the centres coincide. This is particularly clear from the Type II perspective, in which they arise from two different branes on two different two-cycles, and which in general will give two 0-branes at different points in $D = 4$ space-time. If the corresponding two BPS states form a bound state, this will fit into a spin- $3/2$ multiplet that becomes massless, leading to enhanced supersymmetry. If they do not form a bound state, then there is no enhanced supersymmetry. A two-particle state constructed from two vector particles can have spin greater than one, but when they both are massless, there is no new higher-spin gauge symmetry.

The structure of the two-particle Type II spectrum is perhaps clearest from the $D = 8$ perspective, i.e. reducing from $D = 10$ (Type II) or $D = 11$ (M theory) on T^2 or T^3 , respectively. A dyonic membrane in $D = 8$ is obtained from a D-2-brane inside a D-4-brane wrapped on T^2 , or from an M-2-brane inside a M-4-brane wrapped on T^3 . We compactify the $D = 8$ Type II theory on $K3$ and choose the example with a (p, q) membrane wrapping S and a (p', q') membrane wrapping S' . The amount of supersymmetry depends on the “alignment” of the complex structures J, J' of S, S' (which are both supersymmetric and so holomorphic with respect to J, J' , respectively) and of the values of (p, q, p', q') . The $\Lambda_{2,2}$ charge lattice is acted on by $SL(2, \mathbb{Z}) \times SL(2, \mathbb{Z})$, but only the diagonal $SL(2, \mathbb{Z})$ is a symmetry of the theory. It will be convenient to use this $SL(2, \mathbb{Z})$ duality to set the magnetic

⁴In [32] a formula for the degeneracy of states of $D = 4$, $N = 4$ string theory is presented. It is unfortunately not suitable for states with the norm of the charge vectors negative.

charge of the second membrane to zero, $p' = 0$. If p is also zero so that the solution is purely electric, then the configuration will break half the $N = 4$ supersymmetry if J, J' are aligned, so that they are parallel in the ALE limit. This is related by $SL(2, \mathbb{Z})$ duality to a purely magnetic configuration with $q = q' = 0$ arising from two D-4-branes, one wrapping $S \times T^2$ and one wrapping $S' \times T^2$ (together with an infinite number of dyonic partners).

If instead $p' = 0$ and $q = 0$, the configuration will break 1/4 of the heterotic supersymmetry if J, J' are arranged so that they are orthogonal in the ALE limit. This is the set-up that gives, in the coincident limit, the spherically symmetric solution (with naked singularity), fitting into a medium-short multiplet. Acting with $SL(2, \mathbb{Z})$ duality then gives an infinite number of partners to this, all of which become massless (with a corresponding choice of the NS-NS two-form gauge field expectation value on $K3$ [27,33]). As S and S' shrink, the limit gives $SU(2) \times SU(2)$ enhanced gauge symmetry. If the two-cycles intersect as they degenerate, an $SU(3)$ enhanced symmetry results (A_2 singularity) [17]. In each case there are two-particle BPS states.

For the purely electric case, the state with charges $q = q' = 1$, $p = p' = 0$ represents two “W-bosons”, one for the $SU(2)$ associated with S and one for that associated with S' , and there is no bound state. Indeed such a bound state would be problematic for the Higgs mechanism picture of gauge symmetry enhancement. As these states are electrically charged, they can be analysed from the point of view of the perturbative heterotic string. The length-squared of these charge vectors is -4 , so that they are not a physical states of a single heterotic string, but do occur in the two-string spectrum.

Consider now the case in which $p' = 0$ and $q = 0$, together with its S-dual partners. If there is a bound state, then there is an infinite enhancement of supersymmetry when S, S' degenerate. Since there are an infinite number of dyonic states (S-dual to the “W-bosons”) that become massless at the points of enhanced gauge symmetry, this limit cannot be described by a field theory. Thus the infinite number of massless states which preserve 1/4 of the supersymmetry, which would include the infinite number of massless gravitini with dyonic charges, cannot be immediately ruled out. The massless states should then fit into representations of some supersymmetry algebra, and if it were some conventional $N > 4$ algebra, it seems likely that this would require an infinite massless multiplet with arbitrarily high spin. This would be worse than the decompactification limit of a Kaluza-Klein theory, which also has an infinite number of particles becoming massless, but with the maximum spin remaining two. This kind of spectrum might result from the tensionless (null) limit of something like a non-critical superstring theory [34]. Note however, that such dramatic behaviour occurs at, e.g., an $SU(2) \times SU(2)$ enhanced symmetry point, but not at an $SU(2)$ one.

However, it is also possible that the supersymmetry enhancement could be associated with a decompactification limit of the type discussed in [35]. If a string theory in $D + 1$ dimensions is compactified on a circle, masses can be introduced via a Scherk-Schwarz mechanism in such a way that supersymmetry is spontaneously broken [35]. In string theory, the limit in which the circle shrinks to zero size is a decompactification to a T-dual string theory in $D + 1$ dimensions, in which the supersymmetry can be restored [36,37]. From the point of view of the D dimensional theory, there are two points at the boundaries of moduli space ($R = 0$ and $R = \infty$) at which (i) infinite towers of Kaluza-Klein modes become massless and (ii) in addition, extra gravitino multiplets become massless, giving extra

supersymmetry. It is possible that there could be a somewhat analogous supersymmetry enhancement in the cases considered here. Evidence for the appearance of an infinite number massless gravitini at special points in moduli space was also found in the study of BPS states of non-critical strings which appear due to the zero-size instantons of exceptional groups [38].

If there is no bound state a much simpler if less dramatic picture emerges. The “double wrapping states” are all two-particle states and there is no enhancement of supersymmetry, only the by-now familiar picture of gauge symmetry enhancement. It would be interesting to check this directly by studying the quantum mechanics of the two-particle system. However, the fact that this would give a simple consistent picture whereas the alternative requires exotic new physics might seem to make the two-particle picture the most likely interpretation of these results.

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